# Introduzione alle tecniche di ottimizzazione ed esperienze nel laboratorio MilleChili

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# Outline

# MilleChili Lab and aim of the research

## MilleChili Lab

MilleChili Lab was born in february 2009 from a collaboration between *Ferrari SpA* and the *Università degli Studi di Modena e Reggio Emilia*

## Objectives of the laboratory

- Design a novel automotive chassis in view of weight reduction and in fulfilment of given structural performance constraints according to Ferrari standards
- Offer a consulting service supporting Ferrari designers in the field of structural optimization of automotive components

# MilleChili Lab rationale and main tools

## A different approach to design

company know-how and designer experience → systematic design by means of optimization techniques

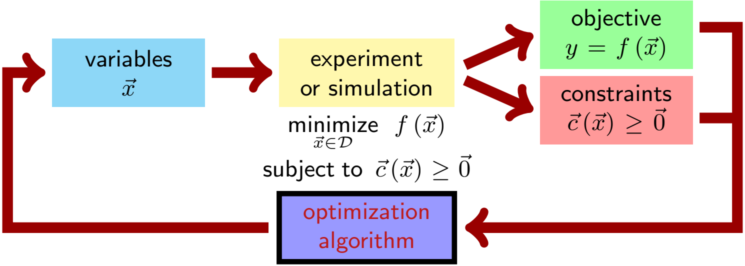
## Structural optimization techniques

- Topology
- Topometry
- Topography
- Size
- Shape

# Optimization

## A definition

In an optimization problem we seek values of the variables that lead to an optimal value of the function that is to be optimized



# Techniques for approaching an optimization problem

## Techniques

- Design of Experiments (DOE)
- Response Surface Modelling (RSM)
- Deterministic Optimization
- Stochastic Optimization
- Robust Design Analysis (RDA)

# Design of Experiments I

## DOE

Is the art of collecting as much information as possible about the problem at hand using the least amount of resources

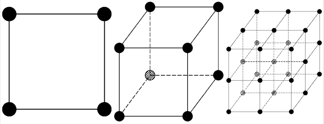
- Technique borrowed from statistics
- In optimization is used for sampling the design space
- The aims of the sampling may be:
    - understand to what extent the variables affect the optimization problem
    - to better reshape the design space for further optimization
    - provide the information for creating a RSM

# Design of Experiments II

## DOE Methods

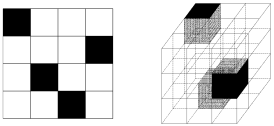
A large set of DOE methods are available, each having different characteristics and being most suitable for specific different purposes, to cite a few:

- Full Factorial



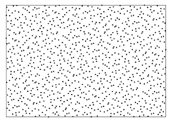
(a) $2^2$ (b) $2^3$ (c) $3^3$

- Latin Hypercube



(a) $k = 2$, $N = 4$ (b) $k = 3$, $N = 3$

- Sobol



(b) Sobol

# Response Surface Modelling I

## RSM

Aims at interpolating or approximating the data collected through DOE in order to create an analytic or parametric response surface (or meta-model) to be used in optimization

$$f\left(\mathbf{x}\right) = \hat{f}\left(\mathbf{x}\right) + \varepsilon\left(\mathbf{x}\right)$$

- Relying on a meta-model is always a risk
- The goodness of the meta-model strictly depends on the quality of the DOE data, and the nature of the function to be approximated
- When collecting data is expensive it is necessary to adopt a DOE+RSM approach in optimization

# Response Surface Modelling II

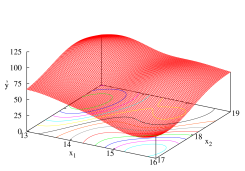
## RSM Methods

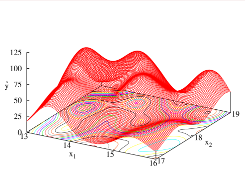
Several RSM methods exists, each having different characteristics and being most suitable for specific different purposes, to cite a few:

- Least-Squares
- Kriging
- Radial Basis Functions

Most methods are weight-based

$$\hat{f}\left(\mathbf{x}\right) = \sum_{i=1}^{N} \lambda_i \left(\mathbf{x}\right) f\left(\mathbf{x}_i\right), \quad \sum_{i=1}^{N} \lambda_i = 1$$



# Deterministic Optimization I

## Deterministic Optimization, AKA Mathematical Programming

Is the classical branch of optimization algorithms in mathematics

- It embodies algorithms that rely on rigorous algebraic methods
- Most algorithms are gradient-based
- The methods converge very fast, and are even faster if the gradients do not need to be approximated with finite-difference methods
- They are local methods
- They are inherently single-objective
- Constraints handling must be carefully addressed, and mis-convergence may easily occur

# Deterministic Optimization II

## Deterministic Optimization Methods

Several deterministic optimization algorithms exist, and they may be classified into a few main categories:

- Newton and Quasi-Newton Methods
- Conjugate Direction Methods
- Non Gradient-Based Methods

Of course each algorithm has its own characteristics that makes it most suitable to address certain types of problems. Nevertheless a few algorithms have imposed themselves for their fast convergence and stability, in particular:

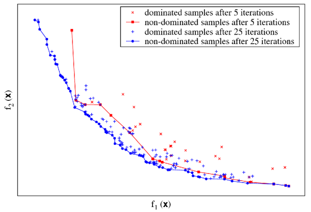
- BFGS (for unconstrained optimization)
- NLPQLP (for constrained optimization)
- Nelder & Mead Simplex (slower but simple and stable)

# Stochastic Optimization I

## Stochastic Optimization

Is the family of optimization methods that are non-rigorous mathematically, and include some stochastic element in their algorithmic formulation:

- Usually the algorithms aim at mimicking aspects of nature in fancy ways
- Randomness allows the samples to move freely in the design space
- This gives the algorithms the chance to better explore the design space, making the optimization global
- The price to pay is in terms of a much slower convergence
- Stochastic algorithms are inherently multi-objective
- The result of a multi-objective optimization is given by the set of non-dominated samples (Pareto front)



# Stochastic Optimization II

## Stochastic Optimization Methods

Among the stochastic optimization methods, a wide popularity has been achieved by:

- Genetic Algorithms
- Evolutionary Algorithms

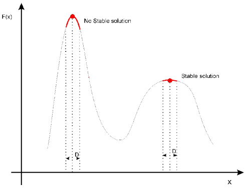
Among the remaining methods, the following are noteworthy:

- Game Theory Optimization
- Particle Swarm Optimization

# Robust Design Analysis I

## Optimum vs Robust

- Finding the optimum configuration is often not enough
- It is important to test the robustness of the solution found
- A solution is said robust if it is little affected by uncertainties




- Robustness is evaluated by including the effect of the variables uncertainties through a Montecarlo sampling in the neighbourhood of a configuration
- The main issue in the robustness analysis is the difficulty in evaluating the uncertainties of the variables

# Robust Design Analysis II

## RDA Methods

Two different kind of approaches to RDA exist:

- Multi-Objective Robust Design Optimization (MORDO)

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{c}(\mathbf{x}) \leq \mathbf{0} \end{array} \quad \Rightarrow \quad \begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \mu(\mathbf{x}) \\ \underset{\mathbf{x}}{\text{minimize}} & \sigma^2(\mathbf{x}) \\ \text{subject to} & \mathbf{c}(\mathbf{x}) \leq \mathbf{0} \end{array}$$

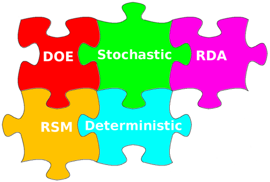
- Reliability Analysis (RA)

  aims at the evaluation of the probability that the configuration will satisfy a given minimum performance requirement

# The Optimization Process

## General Guidelines

- Every class of techniques seen so far and every possible optimization algorithm cannot be thought of as being self-sufficient, and being the solution to any kind of problem
- Every class of techniques/algorithms have their own peculiarities that makes them suitable or unsuitable to properly solve a given problem
- The knowledge of the optimization methods and of the problem at hand must concur in guiding the designer in addressing optimization properly
- All the classes of techniques should be involved in solving an optimization process
- No one-shot approach to optimization should be possibly adopted



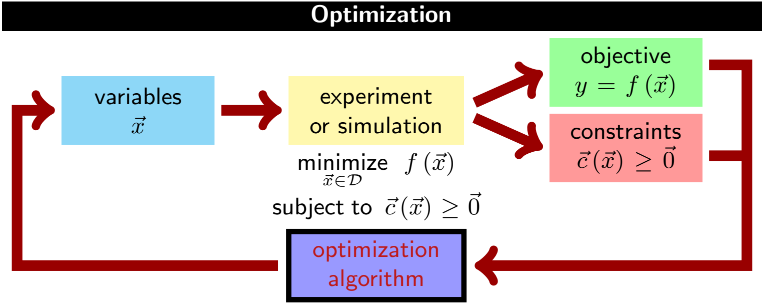
# Structural Optimization: Shape vs Topology

## Shape vs Topology

- When optimizing the shape of an object, we usually resort to a parameterization of the shape, and solve the problem using the techniques proposed before
- Nevertheless, in this way it is not possible to define the optimum topology of the object
- To achieve this a different turn of mind over the optimization problem is needed, and this is the goal of topology optimization

*The art of structure is where to put the holes*

## What is structural optimization?




| Structural optimization | | |
|---|---|---|
| variables (design space) | $\rightarrow$ | FE domain (1 variable per element) |
| experiment or simulation | $\rightarrow$ | finite elements analysis |
| objective | $\rightarrow$ | mass minimization |
| constraints | $\rightarrow$ | stiffness, displacements, modal, ... |
| optimization algorithm | $\rightarrow$ | gradient based (CONLIN or MMA) |

# Variables in structural optimization

| optimization method | variable | applicability |
|---|---|---|
| topology | element density (material distribution) | solid & shell elements |
| topometry | element thickness (thickness distribution) | shell elements |
| topography | element offset (bead patterns) | shell elements |
| size | component thickness (thickness distribution) | shell elements |
| shape | morphing weight factors (deformations superposition) | solid & shell elements |

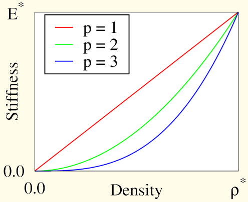
# A quick introduction to topology optimization

## Basic equations: SIMP method for topology optimization

$$x_i \in (0,1] \quad \forall i$$
$$\rho_i(x_i) = x_i \rho^*$$
$$E_i(x_i) = x_i^p E^*$$

$$\Longrightarrow$$

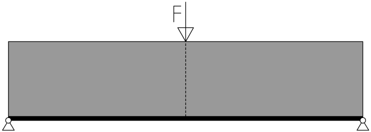
$$\frac{\rho_i(x_i)}{E_i(x_i)} = x_i^{1-p}\frac{\rho^*}{E^*}$$
$$\frac{\partial E_i(x_i)}{\partial x_i} = p x^{p-1} E^*$$

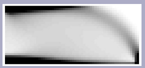
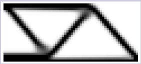
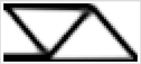



## Main control parameters in topology optimization

- penalty factor $p$
- sensitivity filter $r$

# An example: topology optimization of a bridge




| $p \rightarrow$ <br> $r \downarrow$ | 1.0 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|
| 1.0 |  |  |  |  |
| 1.2 |  |  |  |  |
| 1.5 |  |  |  |  |

images obtained with the code by Sigmund, available from http://www.topopt.dtu.dk/files/top.m

# Application examples

In the following slides a few application examples of structural optimizations related to the automotive chassis weight reduction will be given, in detail:

## Examples and optimizations performed

- Automotive hood

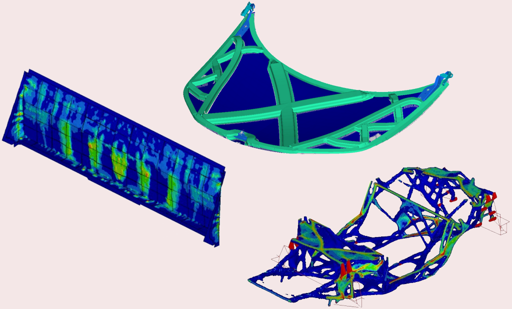
  topology
  ↳ topometry
  ↳ size

- Rear bench

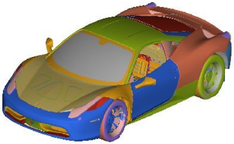
  topography + size

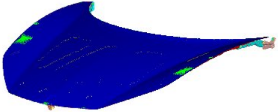
- Automotive chassis

  topology
  ↳ topometry
  ↳ size



# Numerical model, constraints and target





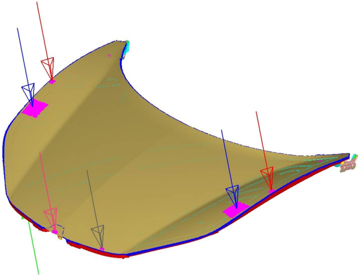
- the reference model is Ferrari F458 Italia
- the objective of the optimization is the hood weight reduction
- the goal has to be reached by modifying the inner panel and its reinforcements with no performance loss compared to the reference model
- the inner panel is connected to the external skin with structural glue
- the outer shape, the hinges and the latch are non modifiable

# Loadsteps

Six different loadsteps have beeen considered in the FE analyses, one for each hood stiffness target

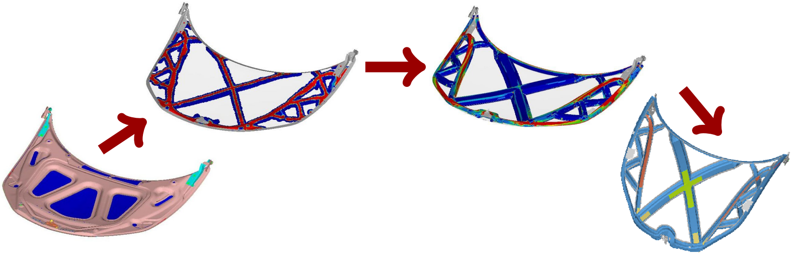
- bending global stiffness
- torsional global stifness
- flaps bending stiffness
- hinges attachment stiffness
- latch attachment stiffness
- stabilus attachment stiffness

FE tests replicate experimental setups



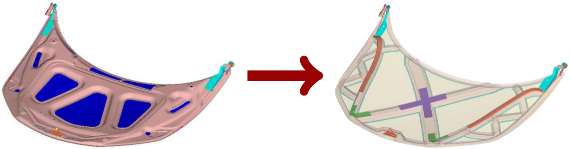
## Optimization process

Topology optimization → Topometry optimization → Size optimization



- the mass distribution proposed by topology optimization is very different from the reference substructure
- topometry optimization locate critical areas needing reinforcements
- size optimization is used for sizing reinforcements granting structural stiffness and weight reduction

# Results

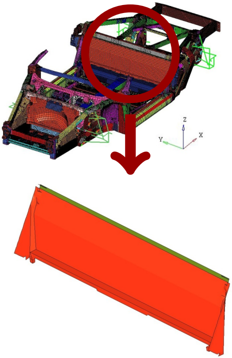


| | |
|---|---|
| global torsional stiffness | − 0.2% |
| global bending stiffness | + 2.4% |
| flaps bending stiffness | + 1.2% |
| hinges attachment stiffness | + 1.2% |
| latch attachment stiffness | + 5.5% |
| stabilus attachment stiffness | +10.9% |

| | |
|---|---|
| hood weight | −12.4% |

# Numerical model

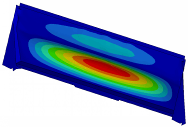
- the rear bench is made of a thin plate of aluminum
- its vibration characteristics are fundamental for passengers comfort
- for this reason the plate is covered with patches of damping material
- the FE property used in this application allows a double material layer to be defined in order to account for the bonding between aluminum and damping material

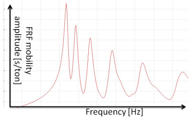


# Targets and optimization process

targets

- first normal mode frequency
- FRFs peaks (related to noise)






the process adopted includes concurrent plate beads pattern and damping material distribution optimizations

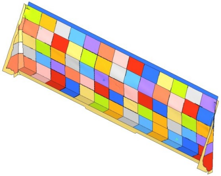
**Size optimization** + **Topography optimization**

damping material thickness $0 \div 3.2\,\mathrm{mm}$

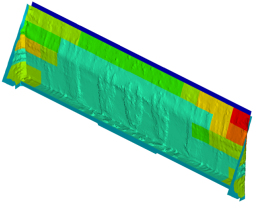
beads distribution on the plate

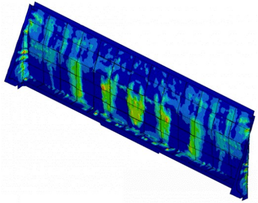
## Variables and results

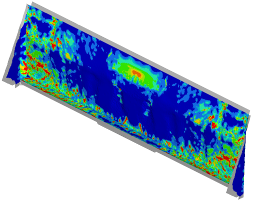
For size optimization a single variable/component is adopted for the aluminum plate thickness, while several variables/components are considered for the damping material



In comparison to the reference model several optimization runs brought to weight reductions in the range 6 to 10%





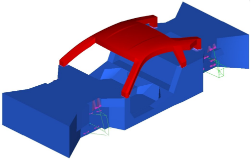


## Topology domains

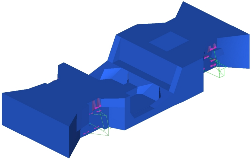
Reference model

- wheelbase and track
- suspensions, seats, engine, and gearbox joints position
- suspensions layout
- passenger compartment, engine, and gearbox size and location
- chassis material (aluminum)

Coupé topology domain



Spider topology domain



# Optimization constraints

1. global bending and torsional stiffness of the structure
   - sills displacement
   - wheel centre displacement
2. head-on crash linearization
   - seat, engine, A-pillar, pedal, flame shield, dashboard displacements
   - compliance
3. modal response of the structure
   - first natural mode
4. local stiffness of the suspensions, engine, and gearbox joints
   - wheel centres displacements
   - engine and gearbox displacements

# Optimization process

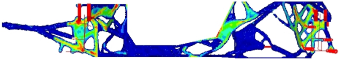
- the chassis is required to be symmetric in the spanwise direction
- increasing complexity optimizations adding the constraints one at a time

Topology optimization → Topometry optimization → Size optimization

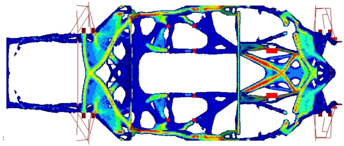
# Results

## Coupé chassis

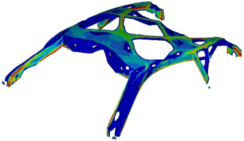
*left view*



*top view*

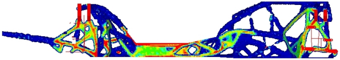


*roof isometric view*

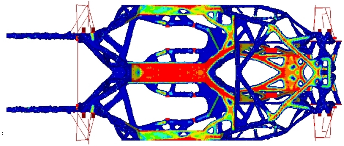


## Spider chassis

*left view*



*top view*



# Conclusions

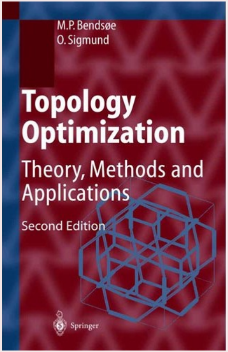
- Structural optimization methods were introduced and application examples discussed showing their potential
- Optimization allows addressing the design process in a more automatic and unitary way
- Structural optimization could provide large benefits to industries by boosting their structural design capabilities
- Its application requires some knowledge and care, particularly in the choice of the domain and the optimization constraints
- Different optimization methods have different features: a combined approach adopting different techniques is recommended for a better exploitation of these features

# Some Bibliography

## On Optimization in General


M. Cavazzuti, *Optimization methods: from theory to design*, Springer, 2013

## On Topology Optimization

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# Thank you for your attention!

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